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## QUADRUPOLE CORE POLARIZATION CHARGES IN THE $A=100$ AND $150$ MASS REGIONS

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The neutron and proton quadrupole core polarization charges are evaluated from the comparison of  $0_{\text{gs}}^+ \rightarrow 2_1^+$  quadrupole transition matrix elements measured by inelastic scattering of different incident probes on Mo, Pd, Cd, Nd, Sm, Gd and Dy isotopes. The resulting values, and in particular those obtained in the framework of the interacting boson model, are discussed with special attention to the evaluation of the strength of low-lying mixed-symmetry states.

The evaluation of effective core polarization charges is a long standing problem in nuclear model calculations. At present they are of particular interest for the prediction of a new class of low-lying states, which are described as isovector ( $T$ -vector) states in geometrical models [1] or as  $F$ -spin-vector states, with a mixed neutron-proton symmetry, in the interacting boson model (IBA) [2]. A low-lying  $1_m^+$  in  $^{156}\text{Gd}$  was the first state of this kind to be observed [3]. A number of studies has been devoted to the search for  $2^+$  mixed-symmetry states ( $2_m^+$ ) [4-6]. The predicted strength and thus the possibility of observing the  $2_m^+$  states depends strongly upon the effective charges which account for the truncation in the model space. In fact the intensity of electromagnetic transitions and the values of the inelastic cross sections for hadronic probes are proportional to the square of the difference of proton and neutron effective charges.

Recently we have obtained evidence [7] for the

existence of  $T(F)$ -vector states in  $^{104,110}\text{Pd}$  and  $^{112}\text{Cd}$ , but not in  $^{146,150}\text{Nd}$  or  $^{150}\text{Sm}$ . Further measurements, the analysis of which is still in progress, seem to confirm this result. The lack of detection of  $2_m^+$  states in the  $A=150$  mass region, could be due to a large fragmentation of the strength or to a strong mixing with scalar components, but also to the effective charge values. The aim of this letter is to present new information on effective charges for Mo, Pd, Cd, Nd, Sm, Gd and Dy nuclei. The values we obtained offer a convincing explanation for the results reported in ref. [7].

The matrix element  $M(E2)^{(i,i')}$ , which describes the excitation of the first  $2^+$  state by the inelastic scattering of an  $i$ -probe, is related to the proton (neutron) matrix elements,  $M_{p(n)}$ , through the normalized ( $b_p^{(i)} + b_n^{(i)} = 1$ ) interaction strengths of the probe with target neutrons and protons:  $M(E2)^{(i,i')} = b_p^{(i)} M_p + b_n^{(i)} M_n$ . At least a pair of probes is needed to determine  $M_p$  and  $M_n$ . The accuracy obtained with

different pairs has been estimated by Bernstein, Brown and Madsen [8]. The best accuracy is obtained with the pairs  $[\pi^-, \pi^+]$  and  $[p, \text{em}]$ ; a lower accuracy with the pair  $[d, \text{em}]$ . Here  $\pi$ ,  $p$ ,  $d$  and  $\text{em}$  refer to pions, low energy protons, deuterons and electromagnetic excitations, respectively. The  $M_p$  values are completely determined by electromagnetic transitions, since  $b_p^{(\text{em})} = 1$  and  $b_n^{(\text{em})} = 0$ . The values given in column 3 of table 1 have been derived from the adopted values of transition probabilities,  $B(E2)$ , given by Raman et al. [9]. The  $M^{(p,p')}$  and  $M^{(d,d')}$  values (columns 4 and 5) have been obtained in part from an analysis of  $p$  and  $d$  scattering experiments, at the incident energies of 30.6 and 50.8 MeV, respectively, carried out in the present study at the KVI

cyclotron in Groningen and in part from our previous experiments [7,10] or from data available in the literature [11–19]. The values given in table 1 are  $L=2$  radial moments of the real part of the transition potentials used in fitting the differential cross sections by coupled channel calculations. The couplings we considered were those between the  $0_{gs}^+, 2_1^+, 0_2^+, 2_2^+$ , and  $4_1^+$  states, in the case of vibrational or transitional nuclei, and between the  $0_{gs}^+, 2_1^+$  and  $4_1^+$  states, in the case of rotational nuclei. Where necessary the published data have been reanalysed to conform with the present analysis. The  $M_n$  values have been derived from the pairs  $[p, \text{em}]$  and  $[d, \text{em}]$ ,

Table 1

Boson numbers and quadrupole matrix elements in  $e\text{ fm}^2$  units. The numbers in parentheses are experimental errors. The numbers after the parentheses in columns 3 and 6 are values calculated with the core polarization charges obtained in the present work (pw in the last column).

Nucleus	$N_\pi - N_\nu$	$M_p$	$M^{(p,p')}$	$M^{(d,d')}$	$M_n$	Ref.
$^{94}\text{Mo}$	1-1	45.1(0.5)49.	53.4(3.2)	48.8(3.5)	53.4(4.1) 49.	pw, [10-13]
$^{96}\text{Mo}$	1-2	52.1(0.5)53.	61.0(3.2)	58.4(3.0)	64.2(3.4) 64.	[10-13]
$^{98}\text{Mo}$	1-3	51.7(0.5)58.	61.0(3.0)	58.0(2.3)	64.2(3.4) 73.	pw, [10-13]
$^{100}\text{Mo}$	2-4	71.8(0.7)75.	81.0(4.2)	77.8(3.0)	84.0(4.5) 90.	pw, [10-13]
$^{102}\text{Ru}$	3-4	80.8(1.0)88.	89.0(5.0)		91.7(4.5) 95.	[10]
$^{104}\text{Pd}$	2-4	73.1(2.4)75.	84.2(4.0)	80.2(5.0)	87.8(4.9) 90.	[7,10]
$^{106}\text{Pd}$	2-5	81.0(2.2)79.	93.8(4.5)	89.3(5.0)	98.0(5.9) 97.	pw, [10,14,15]
$^{108}\text{Pd}$	2-6	87.2(2.3)82.	103. (4.0)		108. (5.9)103.	[11,14,15]
$^{110}\text{Pd}$	2-7	93.3(2.1)85.	113. (5.0)	107. (6.0)	120. (6.9)108.	[7,11,14,15]
$^{106}\text{Cd}$	1-4	64.0(1.6)62.	75.9(5.7)		79.9(7.7) 79.	[10,16]
$^{108}\text{Cd}$	1-5	65.6(1.5)66.	79.9(6.5)		84.7(8.8) 83.	[16]
$^{110}\text{Cd}$	1-6	67.1(1.5)69.	78.4(5.0)	75.5(5.0)	82.6(6.1) 85.	pw, [10,16,17]
$^{112}\text{Cd}$	1-7	71.4(1.4)37.	81.9(4.0)	79.7(5.0)	86.1(5.0) 86.	[7,10,16,17]
$^{114}\text{Cd}$	1-8	74.2(1.3)76.	82.9(6.0)		85.8(8.0) 86.	[16,17]
$^{116}\text{Cd}$	1-7	74.8(1.3)73.	83.7(7.0)		86.7(9.3) 86.	[10,16,17]
$^{142}\text{Nd}$	5-0	52.0(0.8) -	45.0(3.0)		42.7(4.1) -	pw
$^{144}\text{Nd}$	5-1	74.2(2.0)75.	78.0(5.0)		79.3(7.0) 68.	pw
$^{146}\text{Nd}$	5-2	87.2(1.7) -	85.8(5.1)	95.0(6.0)	88.7(6.7) -	[7]
$^{148}\text{Nd}$	5-3	118. (2.)120.	116. (7.)		116. (9.) 115.	pw
$^{150}\text{Nd}$	5-4	165. (1.)159.	165. (10.)	166. (10.)	164. (11.)159.	[7]
$^{144}\text{Sm}$	6-0	51.6(0.8) -	50.2(9.0)		49.7(12.) -	[18,19]
$^{148}\text{Sm}$	6-2	84.9(1.8)83.	83.0(13.)		82.4(17.) 77.	[18,19]
$^{150}\text{Sm}$	6-3	116. (2.) -	110. (7.)	113. (7.0)	108. (10.) -	[7,18,19]
$^{152}\text{Sm}$	6-4	185. (2.)182.	177. (6.)		174. (8.) 177.	[18,19]
$^{154}\text{Sm}$	6-5	208. (1.)191.	206. (15.)		205. (20.)188.	[18,19]
$^{154}\text{Gd}$	7-4	196. (2.)204.	194. (11.)		193. (15.)197.	pw
$^{156}\text{Gd}$	7-5	215. (1.)213.	211. (12.)		210. (16.)208.	pw
$^{158}\text{Gd}$	7-6	224. (1.)222.	221. (13.)		220. (17.)219.	pw
$^{160}\text{Gd}$	7-7	229. (2.)229.	238. (14.)		241. (19.)229.	pw
$^{162}\text{Dy}$	8-7	230. (3.)253.	240. (13.)		243. (18.)250.	pw
$^{164}\text{Dy}$	8-8	237. (1.)260.	260. (15.)	259. (15.)	274. (18.)260.	pw

averaging over different experiments and assuming, as usual at low incident energies [8],  $b_p^{(p)} = 0.25$ ,  $b_p^{(p)} = 0.75$  and  $b_p^{(d)} = b_n^{(d)} = 0.5$ .

The  $M_{p(n)}$  matrix elements are linked to the valence particle matrix elements,  $Q_{\pi(v)}$ , through the effective charges:  $M_p = e_{\pi\pi}Q_\pi + e_{\pi v}Q_v$  and  $M_n = e_{vv}Q_v + e_{v\pi}Q_\pi$ . The first and second letters in the subscripts of the effective charges indicate respectively the core and valence particles (nucleons or bosons). A difficulty in determining empirically the effective charges arises from the fact that four charges must be derived from only two observables:  $M_p$  and  $M_n$ . Because of the charge symmetry of nuclear forces one should have  $e_{\pi\pi} = e_{vv}$  and  $e_{\pi v} = e_{v\pi}$ . However, as argued by brown and Madsen [20], the effective charges for core neutrons can be greater than those for core protons. This effect, mainly due to the neutron excess, must be taken into account in the case of nuclei with different proton and neutron cores. These considerations should hold also for the boson effective charges of IBA calculations. In conformity with the models of ref. [20], one can write the above four charges in terms of two charges and of an additional parameter, which gives the dependence on the neutron excess:  $e_{\pi\pi} = e_p - a\xi$ ,  $e_{vv} = e_p + a\xi$ ,  $e_{\pi v} = e_n - 3a\xi$  and  $e_{v\pi} = e_n + 3a\xi$ , where  $\xi$  stands for  $(N-Z)/A$ . A value of  $a \approx e_n/4$  is predicted by the no-parameter schematic model [20]. The analysis described below has been repeated for different values of  $a$ . The results obtained justify the disregard of the  $a\xi$  terms and, at least for the nuclei here considered, the assumptions  $e_p = e_{\pi\pi} = e_{vv}$  and  $e_n = e_{\pi v} = e_{v\pi}$ .

Relations between the effective charges of different models can be established. Those between nucleon- and boson-effective charges have been, for instance, recently discussed [21]. The  $Q_{\pi(v)}$  matrix elements are evaluated in the framework of specific models; in the IBA the  $Q_{\pi(v)}$  and therefore the  $M_{p(n)}$  values can be expressed in a very simple way in terms of the boson numbers:

$$\begin{aligned} M_p &= f(N_b) (e_p N_\pi + e_n N_v), \\ M_n &= f(N_b) (e_n N_\pi + e_p N_v), \end{aligned} \quad (1)$$

where  $N_\pi$ ,  $N_v$  and  $N_b$  are the proton, neutron and total number of valence bosons ( $N_b = N_\pi + N_v$ ). The function  $f(N_b)$  changes with the dynamical symmetry of the nucleus [2,4,22], and is given by  $(5/N_b)^{1/2}$  in

the U(5) (vibrational limit),  $[(N_b + 4)/N_b]^{1/2}$  in the O(6) (gamma-unstable rotor limit) and  $[(2N_b + 3)/N_b]^{1/2}$  in the SU(3) (deformed rotor limit). For transitional nuclei a realistic evaluation of the  $f(N_b)$  value may be difficult. To overcome this difficulty, the ratio  $e_n/e_p$ , which does not depend on  $f(N_b)$ , can be considered [8,22-24]:

$$\frac{e_n}{e_p} = \frac{M_p N_v - M_n N_\pi}{M_n N_v - M_p N_\pi}. \quad (2)$$

The ratios shown in fig. 1 have been obtained using this relation and the data in table 1. The proton closed shell, assumed to evaluate the  $N_\pi$ , was  $Z=50$  for all  $A \sim 100$  mass nuclei, except for the Mo isotopes. In previous IBA model studies [25,26] a good description of the structure of Mo isotopes was obtained assuming a mixture of  $N_\pi=1, 3$  configurations. However, the  $N_\pi=1$  configuration (shell closure at  $Z=40$ ), is dominant in the ground state band except in the case of  $^{100}\text{Mo}$ . For this nucleus, to avoid the use of a configuration mixing, an  $N_\pi=2$  has been assumed. This choice simulates the results of the analysis of Sambataro and Molnar [25] and is in agreement with the results of Casten systematics [27]. In the Mo-Pd-Cd region a smooth dependence on the

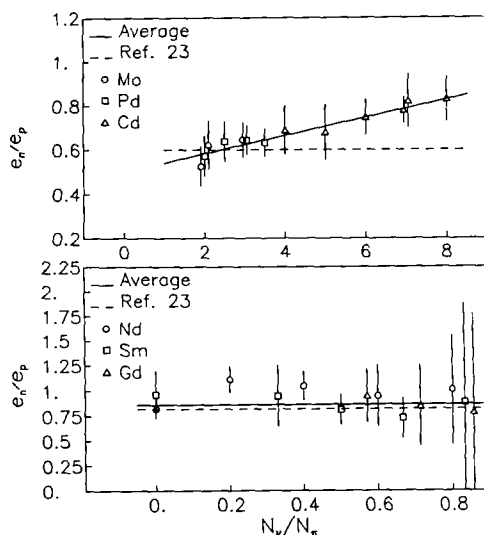


Fig. 1. Ratios of neutron to proton quadrupole effective charges of the IBA model in two different mass regions. The data are plotted against the ratio of neutron to proton valence boson numbers.

ratio  $N_v/N_\pi$  is observed. This dependence is well described by the formula  $e_n/e_p = 0.5 + 0.041 N_v/N_\pi$  (full line in the upper part of fig. 1). In the Nd-Sm-Gd region the dependence on the boson number ratio is less evident. Within the errors we can assume  $e_n/e_p = 0.86$ . For some of these latter nuclei, the accuracy of the method is less satisfactory. It must be noted in fact that, when  $N_\pi = N_v$ , the eqs. (1) and (2) cannot be used and that they have a poor accuracy when  $|N_v - N_\pi|$  becomes small. For this reason  $^{94}\text{Mo}$ ,  $^{160}\text{Gd}$  and  $^{162,164}\text{Dy}$  data were not considered in this first part of the analysis.

The ratios so evaluated are in good agreement with those estimated by Raman et al. [23]. These authors have observed that the experimental  $B(E2)$  values are satisfactorily reproduced, by different IBA and shell model calculations, assuming an effective charge ratio of 0.60 for ( $28 < Z < 50$ ,  $50 < N < 82$ ) and of 0.80–0.82 for ( $50 < Z < 82$ ,  $82 < N < 126$ ). These nuclear regions include those here considered.

An alternative procedure, where applicable, is to assume a given symmetry for some nuclei and to use the eqs. (1) to determine both  $e_p$  and  $e_n$ . Considering for instance the Cd isotopes as vibrational nuclei and assuming therefore  $f(N_b) = (5/N_b)^{1/2}$  we have as average values  $e_p = 21 - N_v/N_\pi$  and  $e_n = 11.2$  in  $e\text{fm}^2$  units. These values are consistent with the above charge ratios. With these charges and using again the eqs. (1) it is possible to deduce the “experimental”  $f(N_b)$  values. It is clear (see fig. 2) that also the Mo, Ru and Pd isotopes follow reasonably well the  $f(N_b)$  dependence predicted in the U(5) case. The Pd isotopes display a slightly less steep  $N_b$  dependence. The disagreement with the U(5) rule is, however, less pronounced than that found by Ginocchio and van Isaker [22], who used the  $M_n$  values obtained from pion scattering [24], which are larger than those estimated from  $[\text{em}, p]$  data. This difference could be due to a more pronounced sensitivity of pions to a neutron rich nuclear surface.

In the higher mass region we have assumed a SU(3) symmetry for  $^{150}\text{Nd}$ ,  $^{152,154}\text{Sm}$  and  $^{154,156,158}\text{Gd}$ . From these nuclei we obtain  $e_p = 14.5 - 2.7 N_v/N_\pi$  and  $e_n = 12.5 - 2.3 N_v/N_\pi$ . The “experimental”  $f(N_b)$  values are shown in the lower part of fig. 2. Also  $^{160}\text{Gd}$  and  $^{162,164}\text{Dy}$  are well described with an  $f(N_b)$  value appropriated to rotational nuclei. The values for  $^{144}\text{Nd}$  and  $^{148}\text{Sm}$  are in agreement with the U(5)

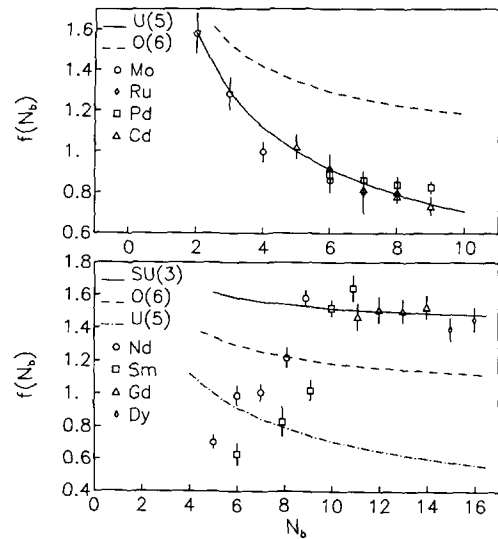


Fig. 2. Experimental and theoretical values of the function  $f(N_b)$ , which depends on the symmetry of the nucleus considered.  $N_b$  is the total number of valence bosons.

curve as suggested by the level spectrum of the same nuclei.  $^{146}\text{Nd}$  and  $^{150}\text{Sm}$  do not have a defined symmetry, whereas  $^{142}\text{Nd}$ , and  $^{144}\text{Sm}$ , which are closed shell nuclei ( $N=82$ ), display very small  $f(N_b)$  values.  $^{148}\text{Nd}$  requires a value near to the O(6) limit. A rather clear indication of the symmetry is therefore obtained for all the nuclei considered, with the exception of  $^{142,146}\text{Nd}$  and  $^{144,150}\text{Sm}$ . Where  $f(N_b)$  is known the  $M_{p(n)}$  values have been calculated using the above effective charges. The resulting values are given by the third numbers in columns 3 and 6 of table 1. They are in agreement within the errors with the experimental values of  $M_n$  and reproduce, with an average accuracy of the order of the 4%, the  $M_p$  values.

We can draw the following conclusions. The boson effective charges have been determined in the lower part of the neutron shells ( $50 < N < 82$ ) and ( $82 < N < 126$ ). In the first the  $e_n/e_p$  ratio shows evidence of a monotonic increase with  $N_v/N_\pi$ . The maximum value is reached in the case of  $^{114}\text{Cd}$ , which lies in the middle of the shell. For Pd isotopes the estimated values are between 0.57 and 0.63, in good agreement with a 0.6 quoted in ref. [23], but larger than the 0.43 estimated by Saha et al. [24]. In the second shell  $e_n$  is of the same order of  $e_p$ , in agree-

ment with the results of refs. [4,23]. The strength for the excitation of  $2_m^+$  states is given by  $M(E2) = (e_n - e_p)(5N_v N_\pi / N_b)^{1/2}$  in the case of U(5) nuclei and by  $M(E2) = (e_n - e_p)[3N_v N_\pi(N_b - 1)]^{1/2} / [(2N_b - 1)N_b]^{1/2}$  for SU(3) nuclei [28]. These formulas and the above effective charges give, for instance, strengths of 17 and  $6 e \text{ fm}^2$  for  $^{110}\text{Pd}$  and  $^{112}\text{Cd}$  respectively, in reasonable agreement with those found in the experiment [7]. In the second shell one obtains strengths of  $2-3 e \text{ fm}^2$ . The resulting cross sections for the excitation of  $2_m^+$  states are of the order of  $10^{-4}$  of those for the excitation of the  $2_1^+$  state, contrarily to a  $10^{-2}$  predicted for nuclei of the first shell. Such small cross sections and the high level densities of  $A \simeq 150$  nuclei can explain the difficulties found in detecting any  $2^+ F(T)$ -vector state in inelastic scattering experiments.

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